Holography and the Cosmic Coincidence

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Based on an analysis of the entropy associated to the vacuum quantum fluctuations, we show that the holographic principle, applied to the cosmic scale, constitutes a possible explanation for the observed value of the cosmological constant, theoretically justifying a relation proposed 35 years ago by Zel'dovich. Furthermore, extending to the total energy density the conjecture by Chen and Wu, concerning the dependence of the cosmological constant on the scale factor, we show that the holographic principle may also lie at the root of the coincidence between the matter density in the universe and the vacuum energy density.

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In spite of the great success of the standard model in describing the evolution and structure of the universe, there are at least two unsolved problems, both related to the existence of a cosmological constant Λ. On the one hand, if Λ originates from vacuum quantum fluctuations, its theoretically expected value has the order of l_p^{-2}, where $l_p \equiv \sqrt{\hbar G/c^3} \approx 10^{-35}$m is the Planck length [1]. That is, 122 orders of magnitude greater than the observed value $\Lambda \approx 10^{-52}\text{m}^{-2}$ [2]. This huge discrepancy is known as the cosmological constant problem.

On the other hand, in an expanding universe with scale factor $a(t)$ (t is the cosmological time), Λ is a constant, while the matter density ρ_m falls with a^3. Therefore, it is an important problem to understand why the observed matter density is so close to the vacuum energy density. This second question is usually referred to as the cosmic coincidence problem.

Another curious coincidence is the so-called large numbers coincidence [3, 4], pointed out by Eddington in the beginning of the 20th century, and which may be expressed in the form

$$\hbar^2 H \approx Gcm^3, \tag{1}$$

where m is a typical hadronic mass, and $H \equiv \dot{a}/a$ is the Hubble parameter.

This surprising relation between physical quantities characteristic of the micro and macrocosmos led Dirac to postulate a possible variation of G with the cosmological time, in such a way that (1) would be valid during the entire evolution of the universe [5]. An alternative explanation, in the spirit of the anthropic principle, was suggested by Dicke, who argued that (1) is not valid for all times, being only characteristic of our era [6].

As we shall see, the large numbers coincidence and the problems related to the cosmological constant are not, in fact, independent problems. They can be related with the help of the holographic principle, postulated some years ago by 't Hooft [7] and Susskind [8]. We will try to show that the application of this principle to the universe as a whole leads to a value for the cosmological constant in accordance with recent observations. Furthermore, with the help of ideas originally proposed by Chen and Wu [9], with respect to the dependence of the vacuum energy density on the scale factor, we will show that the holographic principle may also lie at the root of the cosmic coincidence problem.

In a simplified form, this principle can be described as the extension, to any gravitating system, of the Bekenstein-Hawking formula for black-hole entropy. More precisely, it establishes that the maximum entropy of a gravitating system has the order of A/l_p^2, where A is the area of a characteristic surface which bounds the system, defined in an appropriate manner. In this way, the number of degrees of freedom of the system is not bounded by its volume in Planck units, as expected from quantum theories of space-time, but by the area, in Planck units, of its delimiting surface.

The use of this principle to the universe as a whole depends on the definition of such a surface at a cosmic scale. The existence of a cosmological constant naturally introduces a characteristic surface of radius $\Lambda^{-1/2}$, which may be considered for application of the holographic principle [10]. Let us call this version of the cosmological holographic principle (CHP) as the weak version, for reasons that will soon be clear.

Another version of the CHP [11], referred to hereafter as the strong version, uses as the characteristic bounding surface the Hubble horizon, with radius c/H, for it defines the scale of causal connections for any observer, or what we call the observable universe. It is clear that in an homogeneous and isotropic, infinite (open or flat) universe, filled with a perfect fluid and a positive cosmological constant, the strong version implies the weak one, for the Hubble radius tends asymptotically to $\sqrt{3/\Lambda}$.

In a previous work [4], the CHP is used, in its weak version, as the basis of a possible explanation for the origin of relation (1). To do that, the number of degrees of freedom in the universe is identified with the number of barionic degrees of freedom. Even without entering into the details of the contribution of non-barionic dark matter to the entropy of the universe, we know that this assumption does not have observational basis, since the cosmic background radiation contains about 10^8 photons per barion. Moreover, the major contribution to the entropy of matter seems to come from massive black-holes present in galactic nuclei, which represent an entropy (for

$\kappa_B = 1$) of the order of 10^{101}, while the number of barions in the universe is of the order of 10^{80} [12].

This difficulty can be overcome if we observe that the entropy of the universe is dominated by the vacuum quantum fluctuations that lead, in the classical limit, to the cosmological constant. Following Matthews [13], if we suppose that the vacuum energy density in each cosmological epoch originates from a phase transition with characteristic energy scale mc^2, we can associate to this scale a Compton length $l \approx \hbar/(mc)$. Therefore, the correspondent number of observable degrees of freedom will be of the order of V/l^3, where V is the volume of the observable universe. In other words,

$$N \approx \left(\frac{R}{l}\right)^3 \approx \left(\frac{mc^2}{\hbar H}\right)^3 \tag{2}$$

(R is the Hubble radius).

The last phase transition resulted from quark-hadron confinement, with characteristic energy $mc^2 \approx 150$ MeV [13]. Taking for H the value observed nowadays, $H \approx 65$ km/(sMpc) [2], we obtain $N \approx 10^{122}$, a value that indeed predominates over the entropy of matter given above, of order 10^{101}. Actually, it is easy to verify that (2) dominates the entropy of the universe for any R above the value of decoupling between matter and radiation.

On the other hand, the CHP establishes that the maximum number of degrees of freedom available in the universe is given by

$$N_{max} \approx \left(\frac{R}{l_p}\right)^2 = \left(\frac{c}{Hl_p}\right)^2. \tag{3}$$

If we are restricted to the weak version, we should take for H the asymptotic value $c\sqrt{\Lambda/3}$. In the same way, the maximum value of (2) corresponds to this asymptotic value as well. Identifying both results, we then have

$$\Lambda \approx G^2 m^6/\hbar^4. \tag{4}$$

This expression was derived 35 years ago by Zel'dovich [14], from empirical arguments. It was recently re-derived by Matthews [13], who takes relation (1) as given, as well as the present dominance of the cosmological constant over the density of matter.

As observed by Matthews, (4) is a remarkable relation. Although it is sensible to the parameter m, it leads to correct orders of magnitude, whatever the transition we consider: that of the inflationary epoch, the electroweak transition, or the latest one, due to quark-hadron confinement. In this last case, using $mc^2 \approx 150$ MeV, we obtain $\Lambda \approx 10^{-51}$ m^{-2}, in good agreement with observation, considering that we are not taking into account numerical factors.

Now, if we assume that the present evolution of the universe is dominated by the cosmological constant, as corroborated by observation [2], we can set $H \approx c\sqrt{\Lambda}$, and (4) reduces to (1), the origin of which is then explained. Note that, here, this relation is valid only asymptotically, that is, for times when the cosmological constant dominates. In this respect, the explanation derived via the weak CHP has the same character as the anthropic solution given by Dicke.

On the other hand, if we consider Dicke's solution valid, therefore justifying (1), the result (4) implies the present dominance of the cosmological constant, which can be, in this way, understood [4]. We see therefore that the weak CHP incorporates a new theoretical ingredient to previous approaches, for it represents an independent explanation for relation (4).

Let us consider now the strong version of the CHP. As we saw, after the decoupling between matter and radiation, the entropy (2) associated to the vacuum quantum fluctuations dominates the total entropy of the universe. On the other hand, it is easy to see that (2) represents also the maximum value (for a given R) of the vacuum entropy. Therefore, the strong version of the CHP implies the identity between (2) and (3), leading to $\hbar^2 H \approx Gcm^3$. That is, it leads directly to the large numbers coincidence, eq. (1), without any additional assumption as the dominance of Λ or anthropic arguments.

The price to pay is that, now, (1) is valid for any cosmological time, as originally proposed by Dirac. As a consequence, G will vary with R according to

$$G \approx \frac{(\hbar^2/m^3)}{R} = \left(\frac{\hbar^2}{cm^3}\right) H. \tag{5}$$

This corresponds to the relative variation

$$\dot{G}/G = \dot{H}/H = -(1+q)H, \tag{6}$$

where $q \equiv -a\ddot{a}/\dot{a}^2$ is the deceleration factor. Later on we shall discuss the compatibility of this result with the bounds imposed by observation on a possible time variation of G. For now, let us observe that, in a universe dominated by the cosmological constant, $H \approx c\sqrt{\Lambda}$ is a constant, and $q \approx -1$. In this case, (5) and (6) lead to (4), with G constant, as expected on the basis of the weak version of the CHP.

Let us set aside for now the holographic principle and its cosmological consequences, to introduce some ideas originally proposed by Chen and Wu [9], concerning the dependence of the vacuum energy density on the scale factor. The variation of Λ with a was suggested by those authors as a solution to the problem of the cosmological constant, the value of which would relax with the expansion of the universe. If we associate Λ with the vacuum quantum fluctuations, in the Planck time its value had the order of l_p^{-2}. Therefore, if we assume that Λ varies with a as a power law, such a dependence should have, modulo a constant factor of the order of unity, the form

$$\Lambda \approx l_p^{-2}\left(\frac{l_p}{a}\right)^n. \tag{7}$$

The essential argument pointed out by Chen and Wu is that, if we expect a classical evolution for large times,

there should not be any dependence on the Planck constant in the above relation. In this way we should have $n = 2$, and therefore $\Lambda \approx a^{-2}$. For $a = l_p$, we obtain evidently $\Lambda \approx l_p^{-2}$. But, for the present value $a \approx 10^{-26}$m, one has $\Lambda \approx 10^{-52}\text{m}^{-2}$, in accordance with observation.

The weakness of the above reasoning is that, instead of taking exclusively an explicit dependence on a, as we have done, we can consider also a dependence on, for example, the Hubble radius $R = \dot{a}/a$, or still on the cosmological horizon. The different horizons do not have, in general, the same order of magnitude, and their relation with a depends on the metric, that is, on the complete matter content considered. As we shall see below, this difficulty is naturally overcome when we extend the conjecture of Chen and Wu to the total energy density, as proposed by M.V. John and K.B. Joseph [15].

Let us suppose that the universe starts with a quantum fluctuation at the Planck scale. The matter content generated in this way will have an initial energy density given by $G\rho/c^4 \approx l_p^{-2}$, which, by the same arguments exposed above, will fall with a by the same law (7), with $n = 2$. That is, the total energy density will obey the conservation law

$$\rho a^2 \approx c^4/G. \tag{8}$$

It is easy to verify that, in an isotropic and homogeneous space-time, such a conservation law leads to a linear expansion, with scale factor $a \approx ct$ and $q = 0$. We then have $H \approx c/a$, and so $\rho \approx H^2c^2/G$. Thus, one sees that the total energy density has the order of the critical density, $\rho_c = 3H^2c^2/(8\pi G)$, in accordance with the observed flatness of universe [2].

We see as well that the Hubble radius $R = c/H$ coincides with the scale factor. Therefore, the question of which of the two should be considered for expressing the variation of ρ through equation (7) is irrelevant here. The same can be said with respect to the cosmological horizon, for in an approximately flat universe it has the same order of magnitude as the Hubble horizon.

This is a reasonable result, which shows the internal consistency of this approach. In an isotropic and homogeneous universe born from a quantum fluctuation, the only available parameter, besides the fundamental constants G, c and $\hbar$, is the scale factor, which has a purely geometric nature. Therefore, the matter content so created should lead, necessarily, to causal horizons of the same order of a.

Now, if we treat our matter content as a perfect fluid, the conservation law (8), with G constant, corresponds to the equation of state $p = -\rho/3$. Furthermore, considering it a bi-component fluid [15], composed of dust and of a cosmological constant with energy densities ρ_m and ρ_Λ, respectively, we have

$$\begin{aligned} \rho &= \rho_m + \rho_\Lambda, \\ p &= p_\Lambda = -\rho_\Lambda. \end{aligned} \tag{9}$$

Note that, in equations (9), ρ_m and ρ_Λ do not evolve independently, because of the constraint imposed by the conservation law (8). This constraint leads to a small production of matter (coming from the decaying vacuum energy density), which, as shown in reference [15], is far beyond the current possibilities of detection.

Substituting in relations (9) the equation of state given above, we obtain $\rho_m = 2\rho_\Lambda$. Although a linear expansion, with a null deceleration factor, is compatible with the latest observations [15], such a relation between ρ_m and ρ_Λ is not in accordance with the observational data [2]. As we shall see below, this crucial problem in the approach by John and Joseph can be solved with the help of the holographic principle, leading to results in remarkable agreement with observation.

In a context in which G varies with a, as predicted by the strong version of the CHP, it is possible to show that the conservation law (8) leads as well to a uniform expansion, with $a \approx ct$. However, using (5) (with $R \approx a$), equation (8) turns out to be

$$\rho a \approx c^4m^3/\hbar^2. \tag{10}$$

That is, the total energy density does not fall with a^2, as before, but with a. Therefore, the continuity equation leads now to a new equation of state, given by $p = -2\rho/3$. Substituting in (9), we obtain $\rho_\Lambda = 2\rho_m$. In particular, for $\rho = \rho_c$ this gives $\rho_m/\rho_c = 0.33$, and $\rho_\Lambda/\rho_c = 0.67$.

Note that these are exact results, for they do not depend on numerical factors that we have been neglecting throughout this paper. We thus have an excellent accordance with observation [2]. It constitutes a natural explanation for the cosmic coincidence problem, based on simple and fundamental ingredients: the cosmological holographic principle, in its strong version, and considerations about the conservation of energy in an isotropic and homogeneous universe born from a quantum fluctuation at the Planck scale.

Evidently, expression (4), which gives the absolute value of Λ for a given value of G (and, in particular, the value observed nowadays), remains valid. Indeed, we have seen that $\rho_\Lambda \approx \rho \approx H^2c^2/G$. On the other hand, by definition, $\rho_\Lambda \approx \Lambda c^4/G$. Equating both results, we have $\Lambda c^2 \approx H^2$, which, substituted in (5), leads directly to equation (4).

We should not forget that these results refer to late eras of universe evolution, i.e., after the decoupling between matter and radiation, when the expansion is dominated by a bi-component fluid, composed of dust and of a cosmological constant. For early times, all will depend on the matter content considered. In addition, in early eras the major contribution to the total entropy does not come from vacuum quantum fluctuations, which changes the reasoning developed here.

This problem does not belong to the scope of this article, but let us consider, just as an example, a universe dominated by radiation at temperature T. If we associate to each photon an energy mc^2, and the corresponding Compton length $l \approx \hbar/(mc)$, the maximum number of degrees of freedom remains given, for each time, by

(2). We can then re-obtain (10) by means of the same arguments used before.

But now m is not constant anymore, varying with the temperature as $mc^2 \approx \kappa_B T$, where κ_B is the Boltzmann constant. On the other hand, we know that, for radiation in thermal equilibrium at temperature T, $\rho \propto T^4$. Incorporating both results in equation (10), we obtain $aT =$ constant, or yet $\rho a^4 =$ constant. Thus, we see that the CHP, in its strong version, together with the conjecture by John and Joseph, is consistent with the conservation law for radiation characteristic of the standard model.

This analysis allows us to understand the present dominance of the entropy associated to the vacuum quantum fluctuations over the radiation entropy, which we referred to at the beginning. The former is given by (2), with m constant, i.e., $N_\Lambda \propto (a/\dot{a})^3$. On the other hand, the maximum entropy of radiation, N_γ, is also given at each instant by (2), but with m increasing with T, that is, falling with a. We then have $N_\gamma \propto \dot{a}^{-3}$, which leads to $N_\Lambda/N_\gamma \propto a^3$. Thus, we see that, for small values of a, N_γ dominates, while the contrary occurring for later eras.

This result does not depend on the kind of expansion we are considering (actually, it does not depend on the CHP, nor in the weak nor in the strong versions). But, if the expansion is uniform, as predicted by (8), we see that N_γ will be constant over all the radiation era, as should be for a system in thermodynamic equilibrium.

To conclude, let us finally discuss the compatibility of equations (5) and (6) with the bounds imposed by observation on a possible time variation of G. As we have seen, equation (8) leads to a uniform expansion, with $q = 0$. From (6) we then have, at the present time, $\dot{G}/G = -H \approx -7 \times 10^{-11}$/year.

Recent astronomical observations, generally based on measurements of variations of relative distances in the solar system and in binary pulsars, impose more restrictive limits on the relative variation of G [16]. These results are, thus, in favor of the weak version of the CHP. In this case, relation (4), which explains the observed value of the cosmological constant, would remain valid. The cosmic coincidence problem, however, should wait for another solution.

Nevertheless, let us emphasize that such kind of observations cannot give the ultimate answer to the question. We are considering here a time variation of G over a cosmic scale, in a universe supposed spatially homogeneous and isotropic at the large scale, but not locally. In this context, G is a dynamical quantity that can, at small scales, remain constant, and even suffer spatial variations. Concluding, from local observations, that G does not vary over cosmological scales would be the same as, for example, to conclude, based on observations restricted to our galaxy, that the matter density in the universe is constant at the large scale.

A definite conclusion with respect to this depends on establishing precise limits by means of faithful cosmological observations. Up to our knowledge, such kind of cosmological bounds still are not sufficiently reliable to close the question. In any case, a conclusive study on possible time variations of G will permit deciding between the two versions of the CHP we have been considering.

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[1] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
[2] M. Tegmark, M. Zaldarriaga, and A.J.S. Hamilton, Phys. Rev. D **63**, 043007 (2001).
[3] S. Carneiro, Found. Phys. Lett. **11**, 95 (1998).
[4] G.A. Mena Marugan and S. Carneiro, Phys. Rev. D **65**, 087303 (2002).
[5] P.A.M. Dirac, Nature (London) **139**, 323 (1937).
[6] R.H. Dicke, Nature **192**, 440 (1961).
[7] G. 't Hooft, gr-qc/9310026.
[8] L. Susskind, J. Math. Phys. **36**, 6377 (1995).
[9] W. Chen and Y.S. Wu, Phys. Rev. D **41**, 695 (1990).
[10] R. Bousso, JHEP **0011**, 038 (2000); hep-th/0203101, to appear in Rev. Mod. Phys.
[11] G. Veneziano, Phys. Lett. B **454**, 22 (1999); R. Easther and D. Lowe, Phys. Rev. Lett. **82**, 4967 (1999); N. Kaloper and A. Linde, Phys. Rev. D **60**, 103509 (1999).
[12] R. Penrose, *The Emperor's New Mind* (Oxford University Press, 1989), chapter 7.
[13] R.A.J. Matthews, Astron. & Geophys. **39**, 19 (1998).
[14] Ya.B. Zel'dovich, JETP Lett. **6**, 316 (1967).
[15] M.V. John and K.B. Joseph, Phys. Rev. D **61**, 087304 (2000).
[16] J.G. Williams, X.X. Newhall, and J.O. Dickey, Phys. Rev. D **53**, 6730 (1996); J. Muller *et al.*, Astroph. J. **382**, L101 (1991); T. Damour, G.W. Gibbons, and J.H. Taylor, Phys. Rev. Lett. **61**, 1151 (1988).